PAPER ID:389
DOI: 10.18462/iir.compr.2021.0389

METHOD OF HOUSEHOLD COMPRESSION REFRIGERATION DEVICES ENERGY PERFECTION DETERMINATION AND MATHEMATICAL MODEL OF ITS ESTIMATION

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ABSTRACT

The substitution scheme for a household refrigerator in the form of an equivalent active two-port network, with an evaporator as the input clamps, and a capacitor as the output ones is considered. The mathematical model of the refrigerator as two-port network is developed as a system of two equations of thermal balance with respect to the temperature difference and heat flow rate, which is transformed during refrigerating machine operation. The definitions of idle speed and short circuit modes with regard to the refrigerator are given. The developed model of the refrigerator as two-port network is intended for use in SMART refrigeration systems.

Keywords: Household Refrigerator, Substitution Scheme, Two-port Network, Temperature Difference, Heat Flow Rate, Efficiency, Loading Coefficient

1. INTRODUCTION

Currently, a considerable amount of information has been accumulated on the efficient operation and utilization of household refrigerators under conditions of variable loading of cooling and freezing cabinets. Much of this information is based on experimental data. Using this type of data to control the efficient operation of household refrigerators has always been complicated and has required scientists to use mathematical methods of their simulation. Creation of artificial control systems of the efficient operation of any object, including Smart System in household refrigerator, is only possible if it has an adequate mathematical model.

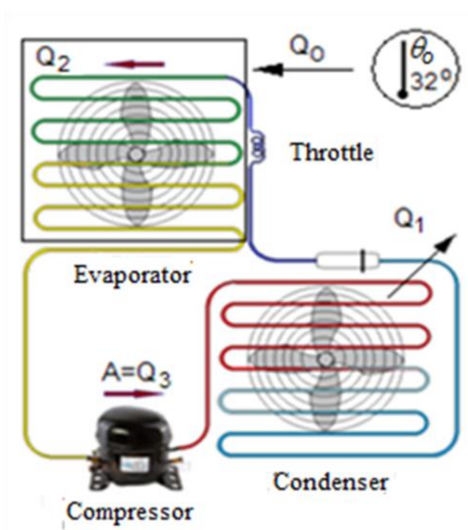

Figure 1: One contour piping system for the refrigerator unit

Such model allows predicting in advance the state of phase variables - heat fluxes, temperatures on separate sections of the refrigerator in the conditions of changing the loading of the useful volume of the cooling cabinets.

The household refrigerator can be considered as a device that provides the capability of excitation in it of the refrigerant aggregate state continuous change process and refrigerant flow in the single-circuit, sealed and closed pipeline system. At the beginning of the simulation, it is possible to consider as indivisible elements of the refrigerator an evaporator with sources of heat flux, a motor-compressor unit with losses of electric energy in the form of heat, a condenser - a heat flux receiver. All these elements are connected by a sealed single-circuit piping system, Fig. 1.

Since the phenomenon of cooling is possible only in case of refrigerant phase state change (depending on the pressure in the pipeline system), motor-compressor unit consumes electricity to create conditions for refrigerant evaporation or condensation. While the refrigerator is turning on, its compressor is providing an increase in the pressure of the refrigerant in the receiver (condenser) and a decrease in the pressure in the source (evaporator), as well as the circulation of the refrigerant in the closed hydraulic circuit. The condition of the phase transition of the working fluid from the gaseous state to the liquid is provided by the condenser and the condition of boiling the working fluid with cooling - by the evaporator. Throttle - capillary or the thermostatic expansion valve sets the required amount of refrigerant in the evaporator depending on the overheating in it (Dossat, 1984), which ensures high performance of the refrigerator as a whole.

2. MAIN SECTION

Mathematical simulation of the household refrigerator technical system is based on the fact that it is composed of sufficiently large and indivisible elements, and the operating mode is determined by phase variables that characterize the physical or informational state of the simulated object. Component equations of large elements establish the relationship of phase variables of different types for each element of the technical system. These are equations of a mathematical model of system elements associated with such phase variables as flow and with variables like potential. The relationship between the homogeneous phases variables related to different elements of the subsystem is established by the topological equations of elements interrelationships. They are obtained on the basis of data on the system structure. As an example of topological equations, equations compiled on the basis of Kirchhoff's laws in electrical systems can be considered. Object simulation should be adaptive to external conditions and should take into consideration their operational changes. Such requirements are met by the versatility of modeling tools, which is ensured by using analogies between component equations in different subsystems.

The phase variables of the domestic refrigerator's thermal subsystem are thermal flows Φ_k and temperature θ_j, which are analogues of currents and voltages in electrical systems. Topological equations should be based on equilibrium equations such as the Kirchhoff laws. The analogue of the first law is the equilibrium equation of heat fluxes in the nodes of the subsystem $\sum_{k\in p}\Phi_k = 0$, of the second law - $\sum_{j\in q}\tau_j = 0$, , in which $\tau_j = \theta_0 - \theta_j$ – is the temperature head on a section of the subsystem circuit.

From the Fourier Law equations for thermal conductivity $\psi = \lambda(\theta_0 - \theta_j)/\ell$ and Newton for convection $\psi = \propto_{conv}(\theta_0 - \theta_j)$, where ψ - heat flux density; λ – heat conductivity coefficient; $\propto_{conv}$ - coefficient of heat exchange through convection; $\theta_0 - \theta_j$ - temperature head at the borders of the section under consideration, with length ℓ for an inductive heat exchanger; θ_0 и θ_j - ambient and body temperature for the convective heat exchanger, follows that in order to obtain the phase variable - heat flow, both parts of the equations must be multiplied by the surface area of the heat exchange S of the selected area, i.e. for conductive and convection heat exchange section it is possible to write down

$$\Phi = \left(\frac{\lambda S}{\ell}\right)(\theta_0 - \theta_j) = \frac{(\theta_0 - \theta_j)}{r_{cond}} \quad \text{и} \quad \Phi = \propto_{conv} S(\theta_0 - \theta_j) = \frac{(\theta_0 - \theta_j)}{r_{conv}}.$$

Phase variable multipliers are conductive and convection impedances

$$r_{cond} = \frac{\ell}{\lambda S} \quad \text{и} \quad r_{conv} = \frac{1}{\propto_{conv} S}.$$

2.1. Theoretical Foundation

Based on the Carnot cycle diagram, Dossat (1984), the refrigerating machine can be considered to be composed of two large and indivisible elements: the evaporator - at the inlet, the condenser - at the outlet. The interaction between them is provided by the throttle and the compressor at the four reference points A, B, D, E, Fig**.** 2.

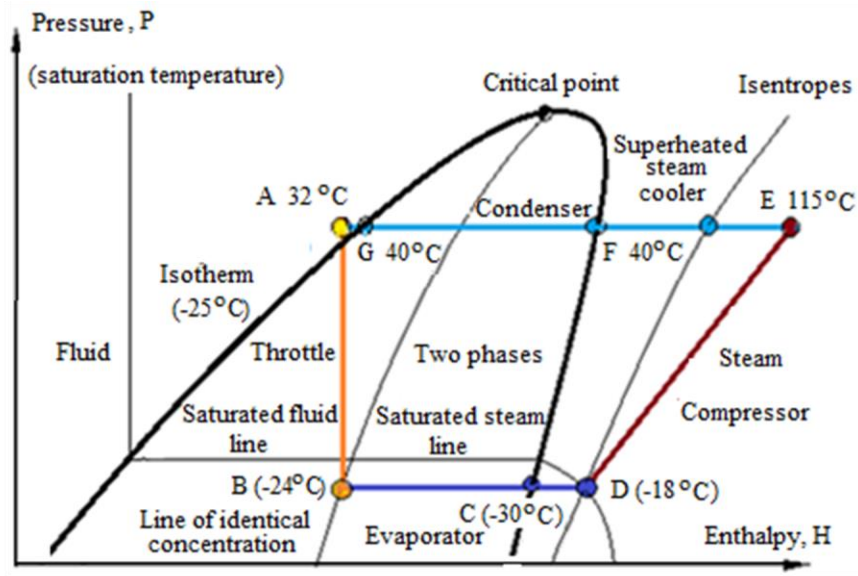

Figure 2: Diagram pressure - enthalpy

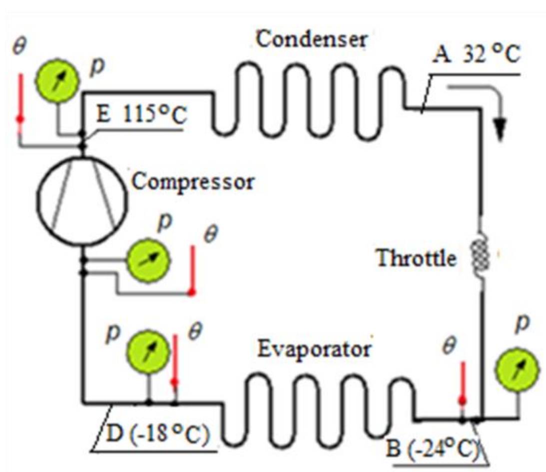

Figure 3: Schematic diagram of the compressor refrigeration performance testing under the simplified methodology

From here, the refrigerating machine has the appearance of a two-port network with two input - B, D and output - D, E clamps, Baidak (2009). They are related to the pressure P and the temperature θ at the inlet-outlet of the compressor unit and the evaporator, as provided for by the test conditions described in European standard (EN28187), Fig**.** 3. According to them, by means of refrigerating machine cycle diagram $ln\, P = f(H)$, and one that corresponds to the type of the used refrigerant, the values of the specific enthalpy H at the inlet and outlet of the evaporator and compressor can be found graphically. According to their values, the specific refrigerating capacity of the evaporator q_0, the specific compression work of the compressor and, accordingly, the mass flow rate of the refrigerant G_a, are calculated. The refrigerating capacity to be determined is calculated as the sum of the product of the values of the evaporator refrigerating capacity and the refrigerant mass flow rate over the entire determined period of the working cycle T, i.e. as $Q_2 = \int_0^T G_a q_0 dt$.

The objective of this work is mathematical simulation of the compression household refrigerator operation mode by the system of two equations made on the basis of U-shaped thermal substitution circuit, and those that colligate the phase variables of heat fluxes and temperature pressures in the evaporator, the motor-compressor unit and the condenser and subsequent determination with its help refrigerating machine energy perfection.

The solution of the problem is based on the method of analogy according to which the difference of potentials $\varphi_0 - \varphi_j$ in the electric circuit section (electric voltage U_{0j}) is the equivalent to the temperature difference $\theta_0 - \theta_j$ (temperature pressure τ_j), the amperage I_j is the equivalent to heat flow Φ_j; the electrical resistance r is the equivalent to the convection resistance $r_{conv_j} = \frac{1}{\propto_{conv_j} S_j}$, in which index j is a mark of belonging to the corresponding area or object: $j = 1$ is the capacitor, $j = 2$- evaporator, $j = 3$ - motor-compressor unit. To simplify the analysis, we neglect the conductive resistance of r_{cond}, considering that the heat capacity of large elements of refrigerating machine in the steady state of operation does not change, and the coefficient of thermal conductivity of metal pipelines in all sections of refrigerant movement is $\lambda \gg 1$.

2.2. U-shaped Electrical Substitution Scheme of Refrigerating Machine

Fig. 4 shows the substitution scheme of refrigerating machine in the form of U-shaped active two-pole network, in which 1'-1 "- input clamps for the steady flow of heat flux Φ_0 from the environment through thermal insulation to the evaporator, 2'-2" - output clamps for remove of heat flow Φ_1 from the condenser into the environment. The diagram also shows: Φ_2- the heat flow drained into the evaporator by the object of cooling and from the outside through the thermal insulation of the refrigerator cabinet; Φ_3 - heat flow drained by the engine of the motor-compressor unit to the condenser and caused by the wastes of electric power; τ_1 - temperature pressure on the section condenser - environment; τ_2 - temperature pressure in the evaporator due to the setpoint of the thermostat (hysteresis); r_1 is the convection resistance between the surface of the condenser S_1 with the coefficient of heat exchange through convection with the environment $\propto_{conv_1}$; r_2, r_0 - similarly for the evaporator and the motor-compressor unit, r - the convection resistance between the cooling object and the evaporator and r^* - the convection resistance between the refrigerator cabinet and the environment.

2.3. Two-pole network equation

According to the electrical balance equations drawn up on the basis of Kirchhoff's law, their counterparts for thermal equilibrium will have the form of equations system, which establishes a link of output variables –namely, temperature pressure phase variables and heat flux in the capacitor, with similar to them input values in the evaporator

$$\begin{cases} \tau_1 = f(\tau_2, \Phi_2) \\ F_1 = f(\tau_2, \Phi_2) \end{cases}$$

For the applied substitution scheme, Fig. 4, these dependencies are established using Kirchhoff's second and first laws as follows:

$$\begin{cases} \tau_1 = r_0(\Phi_2 + \Phi_0 + \Phi_3) + \tau_2 = r_0 \frac{\tau_2}{r_2} + \tau_2 + r_0(\Phi_2 + \Phi_3) = \left(\frac{r_0}{r_2} + 1\right)\tau_2 + r_0(\Phi_2 + \Phi_3) \\ \Phi_1 = \frac{\tau_1}{r_1} + \left(\frac{\tau_2}{r_2} + \Phi_2 + \Phi_3\right) = \left[\left(\frac{r_0}{r_2} + 1\right)\tau_2 + r_0(\Phi_2 + \Phi_3)\right]\frac{1}{r_1} + \left(\frac{\tau_2}{r_2} + \Phi_2 + \Phi_3\right) = \\ \tau_2\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{r_0}{r_1 r_2}\right) + \left(\frac{r_0}{r_1} + 1\right)(\Phi_2 + \Phi_3) \end{cases}$$

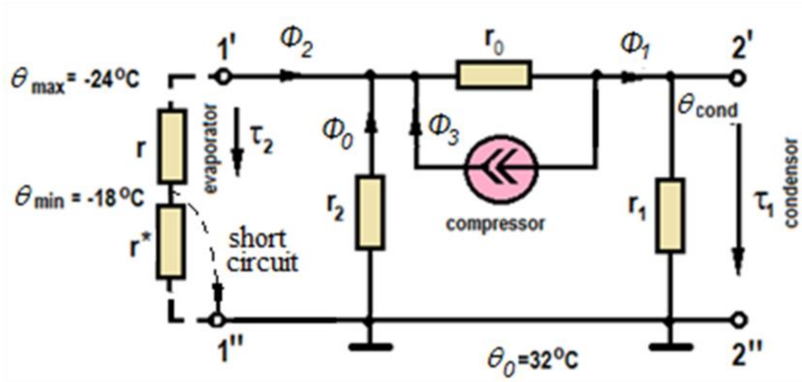

Figure 4: U-shaped electrical substitution scheme of refrigerating machine

Assuming that the obtained equations are considered as invariant for varieties of similar row refrigerators, we introduce the symbols of the coefficients - constants of two-pole network, namely:

$$A = \frac{r_0}{r_2} + 1, \qquad B = r_0, \qquad C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{r_0}{r_1 r_2}, \qquad D = \frac{r_0}{r_1} + 1. \qquad \text{Eq. (1)}$$

Since the system of two-pole network equations is converted to the form:

$$\begin{cases} \tau_1 = A\tau_2 + B(\Phi_2 + \Phi_3) \\ \Phi_1 = C\tau_2 + D(\Phi_2 + \Phi_3) \end{cases} \qquad \text{Eq. (2)}$$

which establishes the relation between the temperature pressure and the heat flow moving in the system of refrigerating machine from the inlet – i.e. from evaporator (τ_2, Φ_2) to the outlet – i.e. to the condenser (τ_1, Φ_1), taking into account the heat inflow from the motor-compressor unit Φ_3 .

2.4. Validation of the Equations of the Active Two-pole Network

Validation of the equation Eq. (2) is made on the example of NORD type refrigerator according to the calculation of which the following indicators are known:
- ambient temperature $t_0 = 32$°C;
- evaporator temperature in freezing mode $t_{max} = -24$°C and defrosting mode $t_{min} = -18$°C;
- heat flow that drains to the sealed refrigeration system from the motor-compressor unit $\Phi_3 = 48\ W$ (accepted at the rated engine power of 120 W with a efficiency of 60%);
- heat flow that enters the sealed system through the evaporator from the cooling objects in the freezing and storage modes $\Phi_2 = 59\ W$;
- the heat flux Φ_0 which enters the evaporator from the environment is neglected due to the large convection resistance of the thermal insulation of the freezer cabinet $r^* > 10$°C$/W$;
- product of heat exchange coefficient due to convection and condenser cooling surfaces $\propto_{conv_1} S_1 = 15 \cdot 0{,}35 = 5{,}25\ W/$°C;

- product of heat transfer coefficient due to convection and evaporator cooling surfaces $\propto_{conv_2} S_2 = 12 \cdot 0{,}35 = 4{,}2\, W/°C$;
- product of the heat exchange coefficient due to convection and cooling surfaces of the motor-compressor unit $\propto_{conv_3} S_3 = 10 \cdot 0{,}25 = 2{,}5\, W/°C$.

The convection resistances correspond to input data of:
- capacitor $r_1 = 0{,}19\, °C/W$;
- evaporator $r_2 = 0{,}238\, °C/W$;
- motor-compressor unit $r_0 = 0{,}4\, °C/W$,

and constants of two-pole network, by Eq. (1):

$$A = \frac{r_0}{r_2} + 1 = \frac{0{,}4}{0{,}238} + 1 = 2{,}68;$$

$$B = r_0 = 0{,}4\, °C/W;$$

$$C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{r_0}{r_1 r_2} = \frac{1}{0{,}19} + \frac{1}{0{,}238} + \frac{0{,}4}{0{,}19 \cdot 0{,}238} = 18{,}31 \frac{1}{°C/W};$$

$$D = \frac{r_0}{r_1} + 1 = \frac{0{,}4}{0{,}19} + 1 = 3{,}105.$$

After the substitution of the two-pole network constants into Eq. (2), the mathematical model of refrigerator takes the form

$$\begin{cases} \tau_1 = 2{,}68\tau_2 + 0{,}4(\Phi_2 + \Phi_3) \\ \Phi_1 = 18{,}31\tau_2 + 3{,}105(\Phi_2 + \Phi_3) \end{cases}. \qquad \text{Eq. (3)}$$

Having heat fluxes, for example, for the NORD-239 type, the temperature pressure on the condenser surface is

$$\tau_1 = 2{,}68 \cdot 6 + 0{,}4 \cdot (59 + 48) = 58{,}9°C,$$

and the heat flow that radiates into the environment from its surface

$$\Phi_1 = 18{,}31 \cdot 6 + 3{,}105(59 + 48) = 442{,}04\, W.$$

According to the obtained values of the distribution of heat fluxes, the efficiency of the household refrigerator is

$$\eta = \frac{\Phi_1 - (\Phi_2 + \Phi_3)}{\Phi_1} = \frac{442{,}04 - (59 + 48)}{442{,}04} = 0{,}76.$$

Performance energy ratio that characterizes the amount of heat subtracted from the cooling object without taking into account the heat input from the environment Φ_0 to the evaporator

$$\varepsilon = \frac{\Phi_1 - (\Phi_2 + \Phi_3)}{\Phi_2 + \Phi_3} = \frac{442{,}04 - (59 + 48)}{59 + 48} = 3{,}13.$$

From the system of two-pole network equations Eq. (2) it follows that the household refrigerator is working in the operating mode, which is between the open (idle) and short circuit operation. The corresponding equations will have the form:

- for open-circuit operation $\Phi_2 = 0$, to which corresponds the absence of the cooling object in the evaporator or its long-term storage without opening the door of the refrigerator cabinet

$$\begin{cases} \tau_{10} = A\tau_2 + B\Phi_3 = 16{,}08 + 19{,}2 = 35{,}28℃ \\ \Phi_{10} = C\tau_2 + D\Phi_3 = 109{,}9 + 149{,}04 = 262{,}9\ W \end{cases};$$

- for short circuit operation with temperature pressure $\tau_2 = \theta_0 - \theta_{max} = 32 - (-24) = 56℃$, to which is corresponding the work of the motor-compressor unit with convection resistance of the cabinet thermal insulation $r^* = 0℃/W$, namely the operation of the refrigerator as an air conditioner in the "winter – summer" mode,

$$\begin{cases} \tau_{1к} = A\tau_2 + B(\Phi_2 + \Phi_3) = 192{,}8\ ℃ \\ \Phi_{1к} = C\tau_2 + D(\Phi_2 + \Phi_3) = 1357{,}44\ W \end{cases}.$$

Based on the linear equations of equilibrium Eq. (2), the loading characteristics of the household refrigerator, namely: $\tau_1 = f(\Phi_2)$ and $\Phi_1 = f(\Phi_2)$ within the operating mode with temperature pressure in the evaporator $\tau_2 = 6℃$, are as shown in Fig. 5.

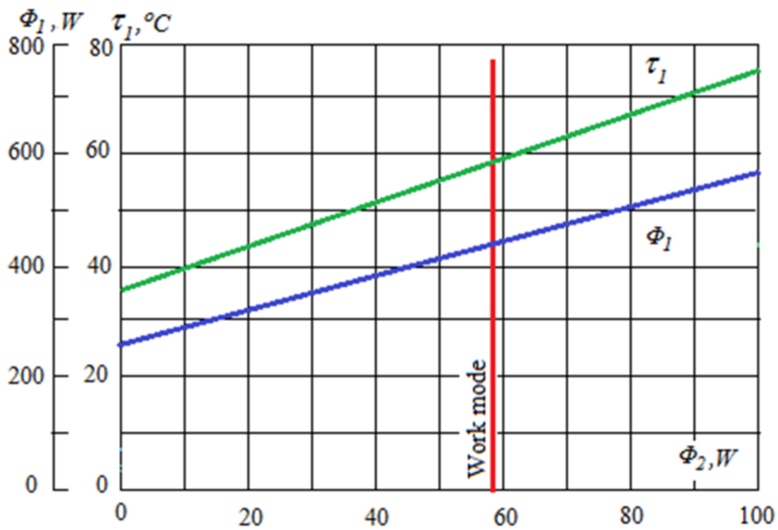

Figure 5: Loading parameters of household refrigerator refrigerating machine $\boldsymbol{\tau_1 = f(\Phi_2)}$ and $\boldsymbol{\Phi_1 = f(\Phi_2)}$

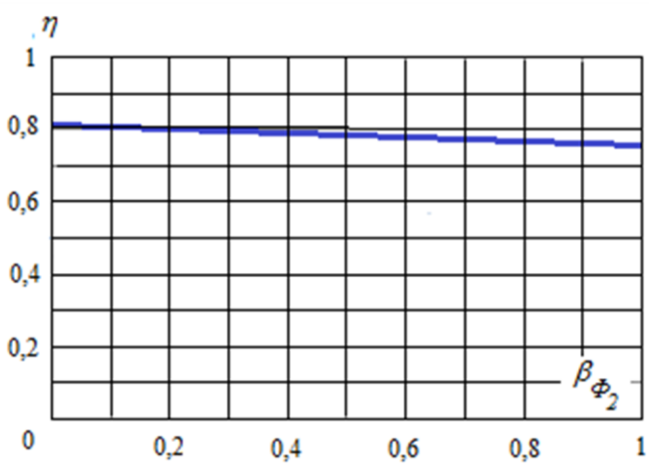

Figure 6: Refrigerating machine efficiency in relation to evaporator loading $\boldsymbol{\eta = f(\beta_{\Phi_2})}$

On the grounds of the obtained system of two-pole network equations, Eq. (3), the efficiency of the household refrigerator, depending on the evaporator loading $\eta = f(\beta_{\Phi_2})$ is

$$\eta = \frac{\Phi_1 - \beta_{\Phi_2}\Phi_2 - \Phi_3}{\Phi_1} = \frac{C\tau_2 + (D-1)\left(\beta_{\Phi_2}\Phi_2 + \Phi_3\right)}{C\tau_2 + D\left(\beta_{\Phi_2}\Phi_2 + \Phi_3\right)}.$$

The dependence of the efficiency of the household refrigerator is shown in Fig. 6. The analysis of the dependence shows that when the loading of the evaporator changes within the nominal value, the efficiency slowly goes down on 5%. At loading $\beta_{\Phi_2} = 0$, the efficiency of the household refrigerator is the highest one and is 0.8 and has been driven by the inflow of heat flow to the condenser only from the motor-compressor unit.

3. CONCLUSIONS

The mathematical simulation of the household refrigerator performed by the above method, and the quantitative assessments obtained in the example, are approximate, but the adequacy of them can be improved by the results of accurate measurements embedded to the equations for values of the heat fluxes, temperature pressures, and convection resistances. The simplicity of the method, the possibility of its application for any refrigerator and the type of mathematical model of household refrigerator in the form of a system of two equations, is an important argument in favor of their application in the systems of refrigerating machine performance regulation and in conducting express analysis of energy efficiency. The substantiation of the boundary modes of operation – open (idling) and short circuit, allows establishing the optimal load of refrigerating machine under the conditions of energy efficient operation of the household refrigerator as a whole. The obtained constants of the active two-pole network are dependent on convection resistances, which enable their variation for the purpose of directed energy efficiency improvement. The chosen scheme for the replacement of the refrigerating machine – by the two-pole network can be improved and provides an opportunity to more adequately evaluation of household refrigerator experimental tests results as well as to reduce the cost of their conduction.

NOMENCLATURE

Φ_k	Heat flux (W)	r_{conv}	convection resistance (°C-1× W-1)
θ_j	temperature (°C)	r_{cond}	conductive resistance (°C-1× W-1)
τ_j	Temperature pressure (°C)	ε	coefficient of performance
$\propto_{conv}$	Heat exchange ratio (W×m-2×°C-1)	β_{Φ_2}	evaporator load factor
S	heat exchange surface (m+2)	r^*	convection resistance of cabinet thermal insulation (°C-1× W-1)

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